Head Office: 2nd Floor, Grand Plaza, Fraser Road, Dak Bunglow, Patna - 01

JEE Main 2023 (Memory based)

31 January 2023 - Shift 1

Answer & Solutions

PHYSICS

1. The ratio of molar specific heat capacity at constant pressure (C_p) to that at constant volume (C_v) varies with temperature (T) as:
[Assume temperature to be low]

A. T^0
B. $T^{1/2}$
C. T^1
D. $T^{3/2}$

Answer (A)

Solution:

We know that:

$$\frac{C_p}{C_v} = \frac{f+2}{f} = \gamma = 1 + \frac{2}{f} = constant$$

We take f to be constant for molecule at low temperature (Independent of temperature)

$$\frac{C_p}{C_v} \propto T^0$$

2. A drop of water of $10\ mm$ radius is divided into 1000 droplets. If surface tension of water surface is equal to $0.073\ J/m^2$ then increment in surface energy while breaking down the bigger drop in small droplets as mentioned is equal to

A. $8.25 \times 10^{-5}\ J$
B. $9.17 \times 10^{-4}\ J$
C. $9.17 \times 10^{-5}\ J$
D. $8.25 \times 10^{-4}\ J$

Answer (D)

Solution:

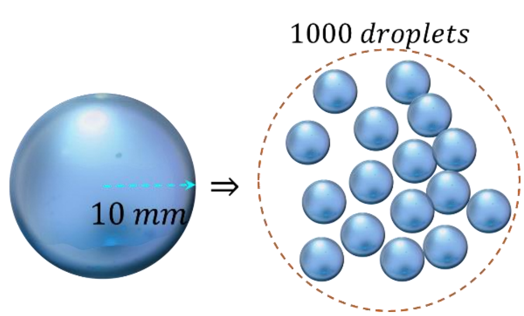
Let the radius of one small droplet is r then:

$$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (10)^3$$

$$\Rightarrow r = 1\ mm$$

$$v_f = 1000 \times 4\pi r^2 T = 1000 \times 4\pi \times 10^{-6} \times 0.073$$

$$v_f = 9.17 \times 10^{-4}\ J$$



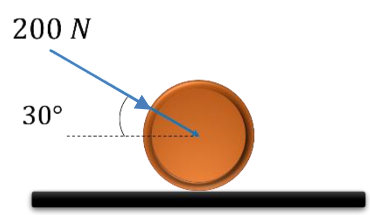
$v_i = 4 \times \pi \times (10^{-2})^2 T = 9.17 \times 10^{-5}\, J$

So,

$\Delta U = 8.25 \times 10^{-4}\, J$

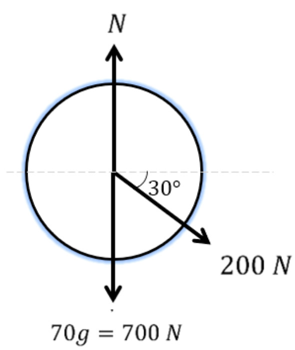
3. A force $200\, N$ is exerted on a disc of mass $70\, kg$ as shown. Find the normal reaction given by ground on the disc.

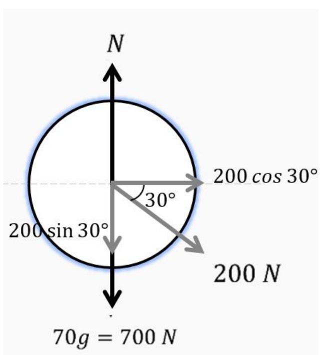
A. $200\, N$
B. $600\, N$
C. $800\, N$
D. $200/\sqrt{3}\, N$



Answer (C)

Solution:





For Equilibrium condition, $\Sigma F_y = 0$
$N = mg + F_{\perp} = 700 + 100 = 800\, N$

4. At depth d from surface of earth acceleration due to gravity is same as its value at height d above the surface of earth. If earth is a sphere of radius $6400\, km$, then value of d is equal to

A. $2975\, km$
B. $3955\, km$
C. $2525\, km$
D. $4915\, km$

Answer (B)

Solution:

Given $g_h = g_d$

We know that:

$$g_0\left(1-\frac{d}{R}\right)=\frac{g_0}{\left(1+\frac{d}{R}\right)^2}$$

$$\left(1+\frac{d}{R}\right)^2\left(1-\frac{d}{R}\right)=1$$

On solving:

$$\frac{d}{R}=0,-\left(\frac{\sqrt{5}+1}{2}\right),\left(\frac{\sqrt{5}-1}{2}\right)$$

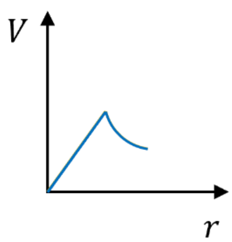
So,

$$d=\left(\frac{\sqrt{5}-1}{2}\right)R$$

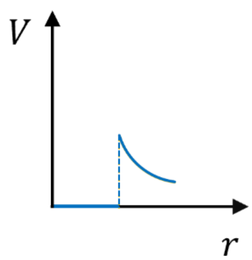
$$d=3955\ km$$

5. Which of the following graphs depicts the variation of electric potential with respect to radial distance from center of a conducting sphere charged with positive charge.

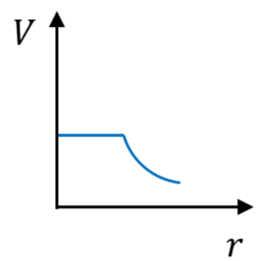
A.



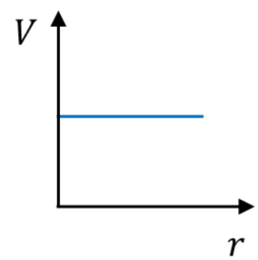
B.



C.



D.

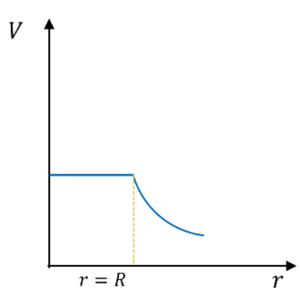


Answer (C)

Solution:

$$V(r)=\begin{cases}\dfrac{q}{4\pi\varepsilon_0 R} & if\ r<R\\ \dfrac{q}{4\pi\varepsilon_0 r} & if\ r>R\end{cases}$$

Where r is the radial distance and R is radius of sphere,
As charge will be on the surface because the sphere is conducting so, graph will be:



6. In a sample of $Hydrogen$ atoms, one atom goes through a transition $n=3\rightarrow ground\ state$ with emitted wavelength λ_1. Another atom goes through a transition $n=2\rightarrow ground\ state$ with emitted wavelength λ_2. Find $\frac{\lambda_1}{\lambda_2}$.

A. $6/5$
B. $5/6$
C. $27/32$
D. $32/27$

Answer (C)

Solution:

Wavelength for transition from $3 \rightarrow Ground\ state$

$$\frac{1}{\lambda_1} = RZ^2\left[1 - \frac{1}{3^2}\right]$$

Wavelength for transition from $2 \rightarrow Ground\ state$

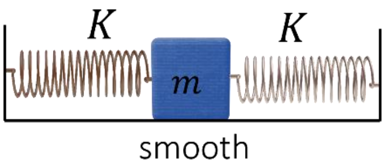
$$\frac{1}{\lambda_2} = RZ^2\left[1 - \frac{1}{2^2}\right]$$

Dividing both equations:

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{3}{4}\right)}{\left(\frac{8}{9}\right)} = \frac{27}{32}$$

7. A block of mass m is connected to two identical springs of force constant K as shown. Find the total number of oscillations of block per unit time.

A. $2\pi\sqrt{\left[\frac{2m}{K}\right]}$
B. $\frac{1}{2\pi}\sqrt{\left[\frac{K}{m}\right]}$
C. $2\pi\sqrt{\left[\frac{m}{2K}\right]}$
D. $\frac{1}{2\pi}\sqrt{\left[\frac{2K}{m}\right]}$



Answer (D)

Solution:

For series combination of springs:

$$K_{eq} = K + K = 2K$$

$$\omega = \sqrt{\frac{K_{eq}}{m}} = \sqrt{\frac{2K}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2K}{m}} \text{ Oscillation per second}$$

8. Consider the two statements:
Assertion: The beam of electrons shows wave nature and exhibits interference and diffraction.
Reason: Davisson - Germer experiment verified the wave nature of electrons.

A. Both are correct. Reason correctly explains assertion.
B. Both are incorrect.
C. Assertion is correct but reason is incorrect.
D. Both are correct. Reason does not explain assertion.

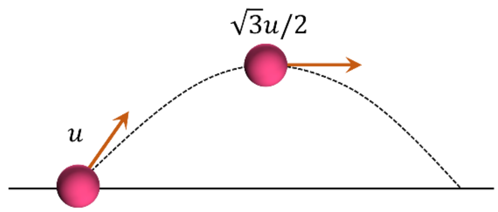
Answer (A)

Solution:

Davisson - Germer experiment verified the wave nature of electrons.

9. A projectile is launched on horizontal surface such that if thrown with initial velocity of u, it has velocity of $\frac{\sqrt{3}u}{2}$ at maximum height. Then time of flight of the projectile is equal to:

A. $\sqrt{3}u/g$
B. $2u/g$
C. u/g
D. $u/2g$



Answer (C)

Solution:

Velocity of ball at maximum height:

$$u\cos\theta = \frac{\sqrt{3}u}{2}$$

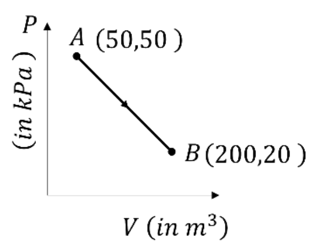
$$\theta = \frac{\pi}{6} \rightarrow angle\ of\ projection$$

Time of flight can be given as:

$$T = \frac{2u\sin\theta}{g} = \frac{u}{g}$$

10. A diatomic gas is taken from point A to point B in a thermodynamic process as described in the pressure–volume graph shown. The change in internal energy is equal to

A. $3.75 \times 10^6\ J$
B. $2.25 \times 10^6\ J$
C. $7.5 \times 10^6\ J$
D. $4.5 \times 10^6\ J$



Answer (A)

Solution:

Change in internal energy
$\Delta U = nC_v\Delta T$

Assuming as to be ideal, $PV = nRT$

$$= \frac{5}{2}(P_fV_f - P_iV_i) \ldots\ldots\ldots.for\ diatomic\ gas, C_v = \frac{5}{2}R$$

$$= \frac{5}{2}(200 \times 20 \times 10^3 - 50 \times 50 \times 10^3)\ J$$

$$= \frac{5}{2} \times 1500 \times 10^3\ J$$

$$= 3.75 \times 10^6\ J$$

11. Unpolarized light of intensity I_o is incident on a polariser A and subsequently on polariser B whose pass axis is perpendicular to that of A . Now a polariser C is introduced between A and B such that pass axis of C is at $45°$ with the pass axis of A. find the intensity of that comes out of B.

A. $\frac{I_o}{8}$

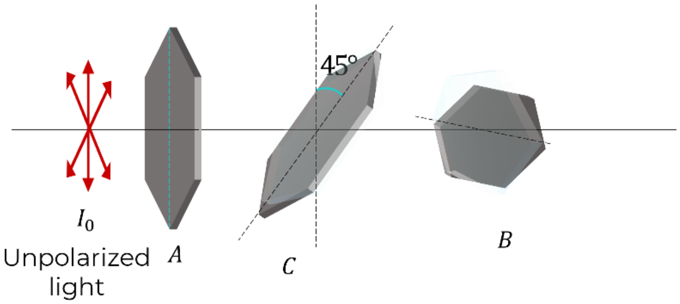
B. $\frac{I_o}{4}$

C. $Zero$

D. $\frac{3I_o}{8}$

Answer (A)

Solution:



Intensity of light passing through A is $I_0/2$

Resultant Intensity can be calculated as:

$$I_{net} = I_0 \times \frac{1}{2} \times \cos^2 45° \times \cos^2 45°$$

12. A bar magnet with magnetic moment of $5\ Am^2$ is lying at stable equilibrium in external uniform magnetic field of strength $0.4\ T$. Work done in slowly rotating the bar magnet to the position of unstable equilibrium is equal to

A. $1\ J$
B. $2\ J$
C. $3\ J$
D. $4\ J$

Answer (D)

Solution:

$$U_i = -MB\cos 0°$$

$$U_f = -MB\cos 180°$$

So,

$$W = \Delta U$$

$$= 2MB = 2 \times 5 \times 0.4$$

$$W = 4\ J$$

13. If n: number density of charge carriers.
A: cross sectional area of conductor
q: charge on each charge carrier
I: current through the conductor
Then the expression of drift velocity is

A. $\frac{nAq}{I}$
B. $\frac{I}{nAq}$
C. $nAqI$
D. $\frac{IA}{nq}$

Answer (B)

Solution:

$$I = nqAv_d$$

$$v_d = \frac{I}{nAq}$$

14. If R, X_L and X_C denote resistance, inductive reactance, and capacitive reactance respectively. Then which of the following options shows the dimensionless physical quantity.

A. $\frac{X_L X_C}{R}$
B. $\frac{R}{\sqrt{X_L X_C}}$

C. $\frac{R}{X_L X_C}$
D. $\frac{R}{(X_L X_C)^2}$

Answer (B)

Solution:

$$X_L = Inductive\ reactance = [R] = dimension\ of\ Resistance$$
$$X_C = Reactive\ reactance = [R] = dimension\ of\ Resistance$$

So, option B, $\frac{R}{\sqrt{X_L X_C}}$ is dimensionless.

15. A conductor of length l and cross-sectional are A has drift velocity v_d when used across a potential difference V. When another conductor of same material and length l but double cross-sectional area than first, is used across same potential difference then drift velocity is equal to

A. $v_d/2$
B. v_d
C. $2v_d$
D. $4v_d$

Answer (B)

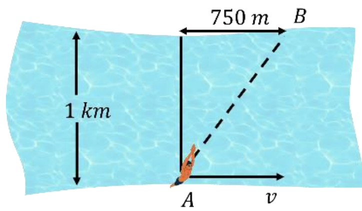
Solution:

$$I = \frac{V}{R} = \frac{V}{\frac{\rho l}{A}} = \frac{VA}{\rho l}$$
$$neAv_d = \frac{VA}{\rho l}$$

All the parameters remain same except cross sectional area and v_d is independent of cross-sectional area when compared in two different conductors so, v_d remains same.

16. A swimmer swims perpendicular to river flow and reaches point B. If velocity of swimmer in still water is $4\ km/h$, find velocity of river flow.

A. $3\ km/hr$
B. $5\ km/hr$
C. $2\ km/hr$
D. $6\ km/hr$

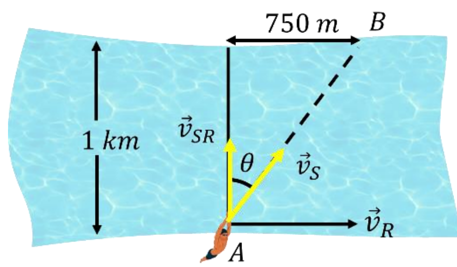


Answer (A)

Solution:

$$\frac{|\vec{v}_R|}{|\vec{v}_{SR}|} = \frac{|\vec{v}_R|}{4} = \tan\theta = \frac{750}{1000}$$

$$|\vec{v}_R| = 3\ km/hr$$



17. A solid sphere is rolling on a smooth surface with kinetic energy $= 7 \times 10^{-3}\, J$. If mass of the sphere is $1\, kg$, then find the speed of the centre of mass in cm/s. (Consider pure rolling)

Answer (10)

Solution:

$$KE = \frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

For pure rolling,

$KE = \frac{1}{2}mV_{cm}^2 + \frac{1}{2} \times \frac{2}{5}mR^2\left(\frac{V_{cm}}{R}\right)^2$ ....$\omega = \frac{V}{R}$ and for solid sphere $I = \frac{2}{5}mR^2$

$$7 \times 10^{-3} = \frac{7}{10}mV_{cm}^2$$

$$V_{cm} = \sqrt{10^{-2}} = 10^{-1}\, m/s = 10\ \ cm/s$$

18. A lift of mass $500\, kg$ starts moving downwards with initial speed $2\, m/s$ and accelerates at $2\, m/s^2$. The kinetic energy of the lift when it has moved $6\, m$ down is__________ kJ.

Answer (7)

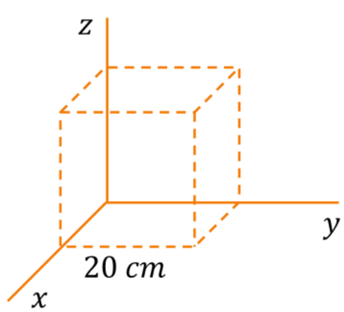
Solution:

$u = 2\, m/s$
$a = 2\, m/s^2$
$s = 6\, m$

For uniform acceleration, $v^2 - u^2 = 2as$
$\Rightarrow v^2 = 2as + u^2 = 2 \times 2 \times 6 + 4 = 28$

So,
$$\text{K.E.} = \frac{1}{2}Mv^2 = \frac{1}{2} \times 500 \times 28 = 7000\, J = 7\, kJ$$

19. Electric field in a region is $4000x^2\, \hat{\imath}\, N/C$. The flux through the cube is $\frac{x}{5}\, Nm^2/C$. Find x.

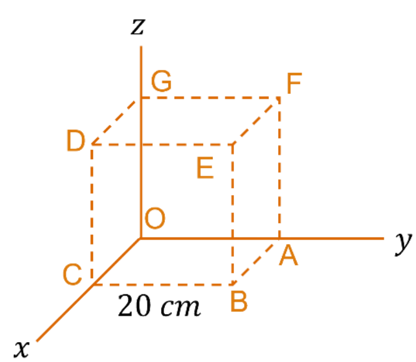


Answer (32)

Solution:

$$\vec{E} = 4000\, x^2\, \hat{\imath}$$

As E_y and E_z are zero,
So, flux through face ABEF, OCDG, EDGF and OABC is zero.

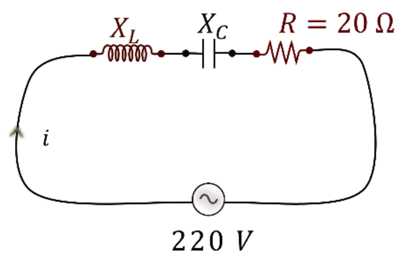


At face OAFG, $x = 0$ so $\phi = 0$
At face EBCD, $x = 0.2\ m$
So,
$\vec{E} = 4000\ x^2\ \hat{\imath} = 4000 \times (0.2)^2 \hat{\imath} = 160\ N/C\ \hat{\imath}$
$\phi_{BEDC} = EA = 160 \times (0.2)^2 = 6.4\ Nm^2/C$
$x = 6.4 \times 5\ Nm^2/C = 32\ Nm^2/C$

20. For a series LCR circuit across an AC source, current and voltage are in same phase. Given the resistance is of $20\ \Omega$ and voltage of the source is $220\ V$. Find current (in A) in the circuit.

Answer (11)

Solution:



The given circuit is in resonance. So,
$i = \frac{V}{R} = \frac{220}{20} = 11\ A$

21. For a particle performing SHM, maximum potential energy is $25\ J$. The kinetic energy (in J) at half the amplitude is $x/4$. find x.

Answer (75)

Solution:

$$\text{Maximum potential energy} = \frac{1}{2}kA^2 = 25\ J$$

$$\text{K. E.} = \frac{1}{2}kA^2 - \frac{1}{2}k\left(\frac{A}{2}\right)^2$$

$$= \frac{1}{2}kA^2\left(\frac{3}{4}\right)$$

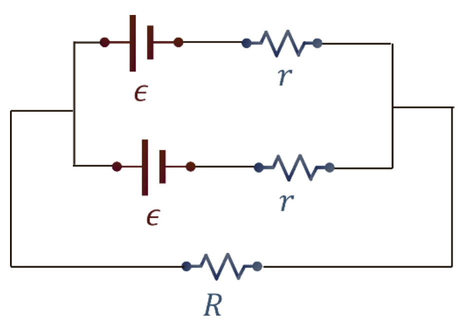
$$= \frac{3}{4} \times 25\ J$$

$$= \frac{75}{4}\ J$$

22. The current through a 5Ω resistance remains same, irrespective of its connection across series or parallel combination of two identical cells. Find the internal resistance $(in\ \Omega)$ of the cell.

Answer (5)

Solution:



When connected in parallel (A):

$$\frac{\varepsilon_{eq}}{\frac{r}{2}} = \frac{\varepsilon}{r} + \frac{\varepsilon}{r}$$

$\varepsilon_{eq} = \varepsilon$ and $r_{eq} = \frac{r}{2}$

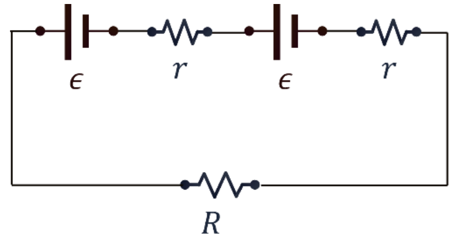
$$current, i_A = \frac{\varepsilon}{R + \frac{r}{2}}$$

When connected in series (B):

$$\varepsilon_{eq} = 2\varepsilon$$

$r_{eq} = 2r$

$current, i_B = \frac{2\mathcal{E}}{R + 2r}$



As, $i_A = i_B$

$\Rightarrow \frac{\mathcal{E}}{R + \frac{r}{2}} = \frac{2\mathcal{E}}{R + 2r}$

$\Rightarrow R + 2r = 2R + r$

$\Rightarrow R = r = 5\Omega$